Enrollment No: ______________________ Exam Seat No: ______________________

C. U. SHAH UNIVERSITY

Summer Examination-2020

Subject Name : Engineering Mathematics - I

Subject Code : 4TE01EMT1 **Branch: B. Tech (All)**

Semester : 1 **Date : 26/02/2020** **Time : 02:30 To 05:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) If $y = \cos\theta + i\sin\theta$, then the value of $y + \frac{1}{y}$ is

(A) $2\cos\theta$ (B) $2\sin\theta$ (C) $2\operatorname{cosec}\theta$ (D) $2\tan\theta$

b) If $\left(\frac{1-i}{1+i}\right)^{100} = a + ib$, then

(A) $a = 2,\ b = -1$ (B) $a = 1,\ b = 0$ (C) $a = 0,\ b = 1$ (D) $a = -1,\ b = 2$

c) If $f(x) = \frac{\log(1+ax) - \log(1-bx)}{x}$ is given to be continuous at $x = 0$, then the value of $f(0)$ must be

(A) $a+b$ (B) $a-b$ (C) $b-a$ (D) $\log\left(\frac{a}{b}\right)$

d) $\lim_{x\to\infty} x^n e^{-x} =$ ______

(A) 0 (B) 1 (C) 2 (D) none of these

e) The interval of convergence of the logarithmic series $log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots\infty$ is

(A) $-1 < x \le 1$ (B) $-1 < x < 2$ (C) $-\infty < x < \infty$ (D) $-1 \le x \le 1$

f) The sum of the series $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + ...$ is

(A) $\log 2$ (B) zero (C) infinite (D) none of these

g) The tangents at the origin are obtained by equating to zero
(A) the lowest degree terms (B) the highest degree terms
(C) constant term (D) none of these

h) If the power of *y* are even, then the curve is symmetrical about
(A) X-axis (B) Y-axis (C) about both X and Y axes
(D) none of these

i) $1+\frac{x^2}{2!}+\frac{x^4}{4!}+\frac{x^6}{6!}+.....\infty$ represent expansion of

(A) $\sinh x$ (B) $\cosh x$ (C) $\cos x$ (D) e^x

j) If $y=\sin^{-1}x$, then x equal to

(A) $1-\frac{y^2}{2!}+\frac{y^4}{4!}-.....$ (B) $1+y+\frac{y^2}{2!}+\frac{y^3}{3!}+.....$

(C) $1-y+\frac{y^2}{2!}-\frac{y^3}{3!}+.....$ (D) $y-\frac{y^3}{3!}+\frac{y^5}{5!}-.....$

k) If $u=ax^2+2hxy+by^2$ then $x\frac{\partial u}{\partial x}+y\frac{\partial u}{dy}$ is equal to

(A) 2u (B) u (C) 0 (D) none of these

l) If $f_1=\frac{vw}{u},\ f_2=\frac{wu}{v},\ f_3=\frac{uv}{w};$ then $\frac{\partial(f_1,f_2,f_3)}{\partial(u,v,w)}$ is equal to

(A) 0 (B) 1 (C) 3 (D) none of these

m) If $Q=r\cot\theta$, then $\frac{\partial Q}{\partial r}$ is equal to

(A) $\cot\theta$ (B) $-\cos ec^2\theta$ (C) $\cot\theta-r\cos ec^2\theta$ (D) $\frac{1}{2}\cot\theta$

n) If $u(x,y,z)=0$ then the value of $\frac{\partial x}{\partial y}\cdot\frac{\partial y}{\partial z}\cdot\frac{\partial z}{\partial x}$ is equal to

(A) 1 (B) -1 (C) 0 (D) none of these

Attempt any four questions from Q-2 to Q-8

Q-2 Attempt all questions (14)

a) Find the continued product of all the values of $\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)^{\frac{3}{4}}$. (5)

b) Evaluate: $\lim_{x\to0}\left(a^x+x\right)^{\frac{1}{x}}$ (5)

c) Evaluate: $\lim_{x\to0}\left(\frac{a}{x}-\cot\frac{x}{a}\right)$ (4)

Q-3 Attempt all questions (14)

a) Using De Moivre's theorem prove that (5)

(i) $\cos5\theta=5\cos\theta-20\cos^3\theta+16\cos^5\theta$

(ii) $\sin5\theta=5\sin\theta-20\sin^3\theta+16\sin^5\theta$

b) Evaluate: $\lim_{x\to0}\frac{e^x+e^{-x}-x^2-2}{\sin^2x-x^2}$ (5)

c) Prove that $\sec h^{-1}(\sin\theta)=\log\cot\frac{\theta}{2}$. (4)

Q-4 Attempt all questions (14)

a) Prove that $(1+x)^x=1+x^2-\frac{1}{2}x^3+\frac{5}{6}x^4-.....$ (5)

b) Expand $\tan^{-1} x$ up to the first four terms by Maclaurin's series. **(5)**

c) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{\tan^{-1} n}{n^2+1}$. **(4)**

Q-5 **Attempt all questions** **(14)**

a) Examine the series $1+\frac{x}{2}+\frac{x^2}{5}+\frac{x^3}{10}+.....+\frac{x^n}{n^2+1}+.....$ for convergence using ratio test. **(5)**

b) Test the convergence of the series $\frac{1}{1\cdot2\cdot3}+\frac{3}{2\cdot3\cdot4}+\frac{5}{3\cdot4\cdot5}+.....$ using comparison test. **(5)**

c) Calculate approximate value of $\sqrt{9.12}$ by using Taylor's theorem. **(4)**

Q-6 **Attempt all questions** **(14)**

a) If $u=\tan^{-1}\left(\frac{x^2+y^2}{x-y}\right)$ then show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{1}{2}\sin 2u$. **(5)**

b) If $u=\log\left(x^3+y^3+z^3-3xyz\right)$ then show that **(5)**

$$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)^2 u=-\frac{9}{(x+y+z)^2}.$$

c) Using Sandwich theorem prove that **(4)**

(i) $\lim_{x\to0} x^2\sin\left(\frac{1}{x}\right)=0$ (ii) $\lim_{x\to0} x^2\cos 20\pi x=0$

Q-7 **Attempt all questions** **(14)**

a) Trace the curve $r=a(1+\cos\theta)$. **(5)**

b) If $x=r\sin\theta\cos\phi,\ y=r\sin\theta\sin\phi,\ z=r\cos\theta$ then find $\frac{\partial(x,\ y,\ z)}{\partial(r,\ \theta,\ \phi)}$. **(5)**

c) Using the formula $R=\frac{E}{I}$, find the maximum error and percentage of error in R if $I=20$ with a possible error of 0.1 and $E=120$ with a possible error of 0.05 and $R=6$. **(4)**

Q-8 **Attempt all questions** **(14)**

a) Trace the curve $xy^2=4a^2(2a-x)$. **(5)**

b) Discuss the maxima and minima of $xy+27\left(\frac{1}{x}+\frac{1}{y}\right)$. **(5)**

c) Discuss the continuity of the function **(4)**

$$f(x,y)=\frac{x}{\sqrt{x^2+y^2}} \text{ when } (x,y)\neq(0,0) \text{ and}$$

$$f(x,y)=2 \qquad \text{when } (x,y)=(0,0).$$